10th Class 2021

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define reciprocal equation.

Ans An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

(ii) Solve: $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Ans Given: $(2x-\frac{1}{2})^2=\frac{9}{4}$

By taking under root both sides, we get

$$\sqrt{(2x-\frac{1}{2})^2}=\sqrt{\frac{9}{4}}$$

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$

$$2x-\frac{1}{2}=\frac{3}{2} \quad ; \quad 2x-\frac{1}{2}=\frac{-3}{2}$$

$$2x=\frac{3}{2}+\frac{1}{2} \quad ; \quad 2x=\frac{-3}{2}+\frac{1}{2}$$

$$=\frac{3+1}{2} \quad ; \quad =\frac{-3+1}{2}$$

$$=\frac{4}{2} \quad ; \quad =\frac{-2}{2}$$

$$2x=2 \quad ; \quad 2x=-1$$

$$\boxed{x=1} \quad ; \quad \boxed{x=\frac{-1}{2}}$$

The solution set will be: $\left\{1,\frac{-1}{2}\right\}$.

(iii) Solve: $\sqrt{3}x^2 + x - 4\sqrt{3} = 0$

Ans $$\sqrt{3}\,x^2 + x - 4\sqrt{3} = 0$$

Compare with $ax^2 + bx + c = 0$

$$a = \sqrt{3}, \quad b = 1, \quad c = -4\sqrt{3}$$

Using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(\sqrt{3}) - (4\sqrt{3})}}{2\sqrt{3}}$$

$$= \frac{-1 \pm \sqrt{1 + 16 \times 3}}{2\sqrt{3}}$$

$$= \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$= \frac{-1 \pm \sqrt{49}}{2\sqrt{3}} = \frac{-1 \pm 7}{2\sqrt{3}}$$

Either $x = \dfrac{-1 + 7}{2\sqrt{3}}$ or $= \dfrac{-1 - 7}{2\sqrt{3}}$

$$= \frac{6}{2\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3}$$

OR $\quad x = \dfrac{-1 - 7}{2\sqrt{3}}$

$$= \frac{-8}{2\sqrt{3}} = -\frac{4}{\sqrt{3}}$$

$\therefore$ Solution Set $= \left\{\sqrt{3}, \dfrac{-4}{\sqrt{3}}\right\}$

(iv) Discuss the nature of roots of equation $3x^2 + 7x - 13 = 0$.

Ans $3x^2 + 7x - 13 = 0$

Here $a = 3$, $b = 7$, $c = -13$

Disc. $= b^2 - 4ac = (7)^2 - 4(3)(-14)$

Disc. = 49 + 168 = 217

Since disc. > 0, and not a perfect square, then the roots are irrational (real) and unequal.

(v) **Find ω^2, if $\omega = \dfrac{-1+\sqrt{-3}}{2}$.**

Ans

$$\omega = \frac{-1+\sqrt{-3}}{2}$$

Squaring

$$(\omega)^2 = \left(\frac{-1+\sqrt{-3}}{2}\right)^2$$

$$= \frac{(-1)^2 + (\sqrt{-3})^2 + 2(-1)(\sqrt{-3})}{2^2}$$

$$= \frac{1 + (-3) - 2\sqrt{-3}}{4}$$

$$= \frac{1 - 3 - 2\sqrt{-3}}{4}$$

$$= \frac{-2 - 2\sqrt{-3}}{4} = \frac{(-1-\sqrt{-3})}{2}$$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

So, if $\omega = \dfrac{-1+\sqrt{-3}}{2}$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

(vi) **Write the quadratic equation having roots: $-1, -7$.**

Ans Sum of the roots:

$$S = \alpha + \beta = -1 + (-7) = -8$$

Product of the roots:

$$P = \alpha\beta = (-1)(-7) = 7$$

Thus the quadratic equation will be:

$$x^2 - Sx + P = 0$$

$$x^2 - (-8)x + 7 = 0$$

$$x^2 + 8x + 7 = 0$$

(vii) If the ratios $3x + 1 : 6 + 4x$ and $2 : 5$ are equal, find the value of x.

Ans $(3x + 1) : (6 + 4x) = 2 : 5$

Product of means = Product of extremes

$$(6 + 4x) \times 2 = (3x + 1) \times 5$$

$$12 + 8x = 15x + 5$$

$$8x - 15x = 5 - 12$$

$$-7x = -7$$

$$x = \frac{-7}{-7}$$

$$x = 1$$

(viii) $a \; \alpha \; \frac{1}{b^2}$ and $a = 3$ when $b = 4$, find a when $b = 8$.

Ans Given $a \; \alpha \; \frac{1}{b^2}$

$\Rightarrow$ $a = \frac{K}{b^2}$ (i)

For $a = 3$, $b = 4$, put in (i)

$$3 = \frac{K}{42}$$

$$3 = \frac{K}{16}$$

$$K = 48$$

Now, put $K = 48$ and $b = 8$ in (i)

$$a = \frac{K}{b^2}$$

$$a = \frac{48}{(8)^2}$$

$$a = \frac{48}{64}$$

$$a = \frac{3}{4}$$

(ix) Find a mean proportional between $x^2 - y^2$, $\frac{x-y}{x+y}$.

Ans Let 'm' be the mean proportional, then

$$x^2 - y^2 : m :: m : \frac{x-y}{x+y} \text{ is in proportion.}$$

We know that
Product of means = Product of extremes

$$m^2 = x^2 - y^2 \times \frac{x-y}{x+y}$$

$$= (x+y)(x-y) \times \frac{x-y}{(x+y)}$$

$$= (x-y)(x-y)$$

$$m^2 = (x-y)^2$$

$$\sqrt{m^2} = \pm\sqrt{(x-y)^2}$$

Taking square root,

$$m = \pm(x-y)$$

3. Write short answers to any SIX (6) questions: (12)

(i) Define a rational fraction.

Ans An expression of the form $\frac{N(x)}{D(x)}$ where N(x) and D(x) are polynomials in x with real coefficient and $D(x) \neq 0$ is called a rational fraction. For example,

$$\frac{x^2+3}{(x+1)^2(x+2)} \text{ and } \frac{2x}{(x-1)(x+2)} \text{ are rational fractions.}$$

(ii) Resolve into partial fractions: $\frac{x-11}{(x-4)(x+3)}$.

Ans $$\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

$$x - 11 = A(x+3) + B(x-4) \qquad \text{(i)}$$

Put $x = 4$, $x = -3$ in (i)
Firstly,

$$4 - 11 = A(4+3) + B(4-4)$$

$$-7 = A(7) + 0$$

$$\Rightarrow \quad 7A = -7$$

$$\boxed{A = -1}$$

And

$$-3 - 11 = A(-3 + 3) + B(-3 - 4)$$
$$-14 = 0 + B(-7)$$
$$\Rightarrow \quad -7B = -14$$
$$\boxed{B = 2}$$

So,

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{-1}{x - 4} + \frac{2}{x + 3}$$

(iii) State De Morgan's laws.

Ans De-Morgan's Laws are:

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

(iv) If X = {1, 4, 7, 9}, Y = {2, 4, 5, 9}, then find X ∩ Y.

Ans

$$X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$$
$$X \cap Y = \{4, 9\}$$

(v) If A = N, B = W, then find B – A.

Ans Now,

$$A = \{1, 2, 3, ...\} = N$$
$$B = \{0, 1, 2, 3, ...\} = W$$
$$B - A = \{0, 1, 2, 3, ...\} - \{1, 2, 3, ...\}$$
$$B - A = \{0\}$$

(vi) Find a and b if (3 – 2a, b – 1) = (a – 7, 2b + 5)

Ans By comparing,

$$a - 7 = 3 - 2a$$
$$a + 2a = 3 + 7$$
$$3a = 10$$
$$a = \frac{10}{3}$$

and

$$2b + 5 = b - 1$$
$$2b - b = -1 - 5$$
$$b = -6$$
$$\therefore \quad a = \frac{10}{3} \quad \text{and} \quad b = -6$$

(vii) Define mode.

Ans Mode is defined as the most frequent occurring observation in the data. It is the observation that occurs maximum number of times in the given data. The following formula is used to determine mode:

For ungrouped data:

Mode = the most frequent observation

For grouped data:

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

(viii) What is a histogram?

Ans A Histogram is a graph of adjacent rectangles constructed on XY-plane. It is a graph of frequency distribution. Both discrete and continuous frequency distributions are represented by histrogram.

(ix) Define standard deviation.

Ans Standard deviation is defined as the positive square root of the mean of the square deviations of $X_i(i = 1, 2, \ldots n)$ observations from their arithmetic mean.

Symbolically,

$$\text{S.D}(X) = S = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n}}$$

4. Write short answers to any SIX (6) questions: (12)

(i) Define ratio and give one example.

Ans A relation between two quantities of the same kind (measured in same unit) is called **ratio**. If a and b are two quantities of the same kind and b is not zero, then the ratio of a and b is written as a : b or in fraction $\frac{a}{b}$.

e.g., if a hockey team wins 4 games and losses 5, then the ratio of the games won to games lost is 4 : 5 or in fraction $\frac{4}{5}$.

(ii) **Find the third proportional to 28 and 4.**

Ans Let c be the third proportional

Then $28 : 4 :: 4 : c$ are in proportion.

We know that,

Product of extremes = Product of means

$$28\,c = 4 \times 4$$

$$28\,c = 16$$

$$c = \frac{16}{18}$$

$$c = \frac{4}{7}$$

So, third proportional is $\frac{4}{7}$.

(iii) **Express 315.18° into D°, M′ and S″ form.**

Ans 315.18°

$$= 315^\circ + (0.18 \times 60)'$$
$$= 315^\circ + 10.8'$$
$$= 315^\circ + 10' + 0.8'$$
$$= 315^\circ + 10' + (0.8 \times 60)''$$
$$= 315^\circ + 10' + 48''$$
$$\therefore \quad 315.8 = 315^\circ\, 10'48''$$

(iv) **Convert $\frac{7\pi}{8}$ into degree.**

Ans $\frac{7\pi}{8}$ rad $= \frac{7\pi}{8}$ (1 radian)

$$= \frac{7\pi}{8}\left(\frac{180^\circ}{\pi}\right) = \left(\frac{7 \times 180}{8}\right) = \left(\frac{315}{2}\right)^\circ$$
$$= (157.5)^\circ = 157^\circ + 0.5^\circ$$
$$= 157^\circ + 30^1 = 157^\circ 30^1$$

(v) **Prove that: $(1 - \sin^2\theta)(1 + \tan^2\theta) = 1$**

Ans L.H.S. $(1 - \sin^2\theta)\,(1 + \tan^2\theta)$

Using $1 - \sin^2\theta = \cos^2\theta$

and $1 + \tan^2\theta = \sec^2\theta$

$$= \cos^2\theta \times \sec^2\theta$$

and $\quad \sec\theta = \dfrac{1}{\cos\theta}$

$$= \cos^2\theta \times \frac{1}{\cos^2\theta}$$

$$1 = \text{R.H.S}$$

$\therefore \quad (1 - \sin^2\theta)(1 + \tan^2\theta) = 1$

(vi) Find θ, when $l = 4.5$ m, $r = 2.5$ m

Ans We know $l = r\theta \Rightarrow \theta = \dfrac{l}{r}$

$$\theta = \frac{4.5}{2.5} = 1.8 \text{ radians}$$

(vii) Express the angle into radian: 135°

Ans We know

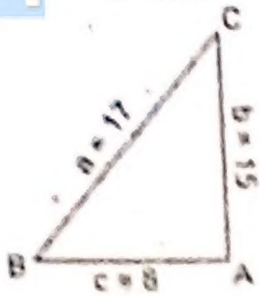
$$180^\circ = \pi \text{ radians}$$

$$1^\circ = \frac{\pi}{180} \text{ radians}$$

$$135^\circ = \frac{\pi}{180} \times 135 = \frac{3}{4}\pi \text{ radians}$$

(viii) In a ΔABC, a = 17 cm, b = 15 cm and c = 8 cm, find m∠B.

Ans for $\quad a = 17$ cm, $b = 15$ cm and $c = 8$ cm



Pythagora's theorem

$$a^2 = b^2 + c^2$$

$$(17)^2 = (15)^2 + (8)^2$$

$$289 = 225 + 64$$

$$289 = 289$$

$\therefore$ ABC is a right angled triangle.

So, $\tan\angle B = \dfrac{\text{Opp. side}}{\text{Adj. side}}$

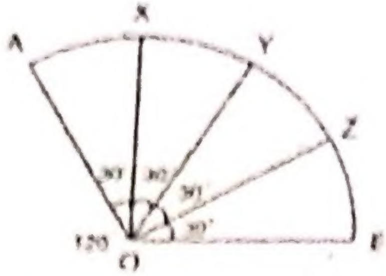
$$\tan B = \frac{15}{8}$$

$$B = \tan^{-1}\frac{15}{8} = 61.9^\circ$$

(ix) Divide an arc of any length into four equal parts.

Ans **Steps of construction:**

(i) Divide an arc AB. The central angle of arc is 120°.



(ii) Divide 120° central angle into four equal parts each of size 30°.

(iii) Produce these angles met AB at point A, X, Y, Z and B.

(iv) Arc AB has been divided into four equal parts.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$$7x^2 + 2x - 1 = 0$$

Ans For Answer see Paper 2019 (Group-II), Q.5.(a).

(b) Solve by using synthetic division, if 3 and −4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$. (4)

Ans

3	1	2	−13	−14	24
		3	15	6	−24
	1	5	2	−8	0
−4		−4	−4	+8	
	1	1	−2	0	

The depressed equation is factorization of

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x + 2) - 1(x + 2) = 0$$
$$(x + 2)(x - 1) = 0$$

Either $x + 2 = 0$

$\Rightarrow$ $x = -2$

OR $x - 1 = 0$

$\Rightarrow$ $x = 1$

$\therefore$ 3, −4, −2, and 1 are the roots of the equation.

Q.6.(a) Using theorem of componendo-dividendo find the value of : $\dfrac{x-3y}{x+3y}-\dfrac{x+3z}{x-3z}$ if $x=\dfrac{3yz}{y-z}$. (4)

Ans Given $\quad x=\dfrac{3yz}{y-z}$

or $\quad \dfrac{x}{3y}=\dfrac{z}{y-z}$

Applying componendo-dividendo theorem,

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$\frac{x+3y}{x-3y}=\frac{z+y-z}{z-y+z}$$

$$\frac{x+3y}{x-3y}=\frac{y}{2z-y}$$

Reversing,

$$\frac{x-3y}{x+3y}=\frac{2z-y}{y} \qquad \text{(i)}$$

Again $\quad x=\dfrac{3yz}{y-z}$

$$\Rightarrow \quad \frac{x}{3z}=\frac{y}{y-z}$$

Applying componendo-dividendo theorem,

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z} \qquad \text{(ii)}$$

Subtracting eq (ii) from eq (i), we get

$$\frac{x-3y}{x+3y}-\frac{x+32}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$= \frac{2z^2 - yz - 2y^2 + yz}{yz}$$

$$= \frac{2z^2 - 2y^2}{yz} = \frac{2(z^2 - y^2)}{yz}$$

$$\therefore \quad \frac{x - 3y}{x + 3y} - \frac{x + 3z}{x - 3z} = \frac{2(z^2 - y^2)}{yz}$$

(b) Resolve into partial fractions: $\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)}$. **(4)**

Ans For Answer see Paper 2020 (Group-II), Q.6.(b).

Q.7.(a) If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9}, B = {1, 4, 7, 10}, then prove that $(A - B)' = A' \cup B$ (4)

Ans For Answer see Paper 2020 (Group-II), Q.7.(a).

(b) Find the standard deviation of five teachers' salaries in rupees: (4)

11,500, 12,400, 15,000, 14,500, 14,800

Ans 11,500, 12,400, 15,000, 14,500, 14,800

$$\bar{X} = \frac{\Sigma X}{n}$$

$$\Sigma x = 11,500 + 12,400 + 15,000 + 14,500 + 14,800$$
$$= 68,200$$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{68,200}{5}$$
$$= 13,640$$

X	$X - \bar{X}$	$(X - \bar{X})^2$
11,500	–2,140	4579600
12,400	–1,240	1537600
15,000	1,360	1849600
14,500	860	739600
14,800	1,160	1345600
68,200	0	10052000

Using $$S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$$

$$= \sqrt{\frac{10052000}{5}} = \sqrt{2010400}$$

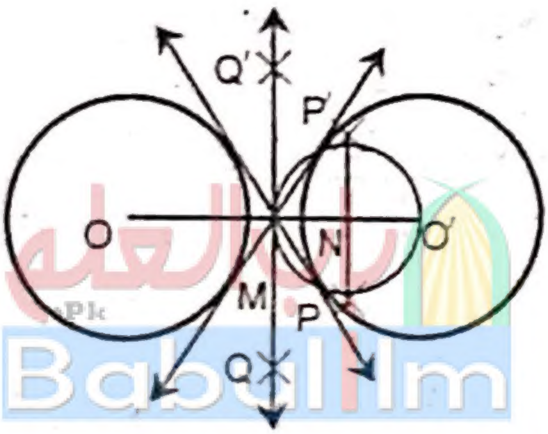
$$= 1417.88$$

Q.8.(a) Prove that: $\dfrac{1+\sin\theta}{1-\sin\theta} - \dfrac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$. **(4)**

Ans For Answer see Paper 2017 (Group-II), Q.8.(a).

(b) Draw two equal circles of each radius 2.4 cm. If the distance between their centres is 6 cm, then draw their transverse tangents. (4)

Ans



Step of Constructions:

(i) Draw $m\overline{OO'} = 6$ cm.

(ii) Draw 2 circles of 2.4 cm radius on O and O'.

(iii) Find M the mid-point of $\overline{OO'}$.

(iv) Draw N, the mid-point of $\overline{O'M}$.

(v) Draw a circle with centre at N and of radius $\overline{O'N}$. This circle intersects the circle at P and P'.

(vi) Join P' with M and produce, it touches the 2nd circle at Q'.

(vii) Join P with M and produce it touches the 2nd circle at Q.

(viii) P'Q' and PQ are the required tangents.

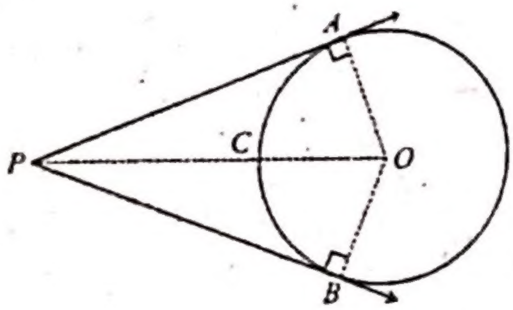
Q.9. A straight line drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. (9)

Ans For Answer see Paper 2021 (Group-I), Q.9.

OR

Two tangents drawn to a circle from a point outside it, are equal in length.

Ans



Given:

Two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P to the circle with centre O.

To Prove:

$m\overline{PA} = m\overline{PB}$

Construction:

Join O with A, B and P, so that we form $\angle rt\Delta^s$ OAP and OBP.

Proof:

Statements	Reasons
In $\angle rt\Delta^s$ OAP $\leftrightarrow$ OBP	
$m\angle OAP = m\angle OBP = 90^\circ$	Radii $\perp$ to the tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$
hyp. $\overline{OP}$ = hyp. $\overline{OP}$	Common
$m\overline{OA} = m\overline{OB}$	Radii of the same circle
$\therefore$ $\Delta OAP \cong \Delta OBP$	In $\angle rt\ \Delta^s$ H.S $\cong$ H.S
Hence, $m\overline{PA} = \overline{PB}$	